# Precision calculation for $e^+e^- \to 2f$: the $\mathcal{KK}$ MC project*

B.F.L. Ward[a] and S. Jadach[b] and Z. Was[b]

[a]Department of Physics and Astronomy, University of Tennessee,
Knoxville, Tennessee 37996-1200, USA

[b]Henryk Niewodniczanski Institute of Nuclear Physics,
ul. Radzikowskiego 152, 31-342 Cracow, Poland

We present the current status of the coherent exclusive (CEEX) realization of the YFS theory for the processes in $e^+e^- \to 2f$ via the $\mathcal{KK}$ MC. We give a brief summary of the CEEX theory in comparison to the older (EEX) exclusive exponentiation theory and illustrate recent theoretical results relevant to the LEP2 and LC physics programs.

UTHEP-02-0901
Sept, 2002

## 1. Introduction

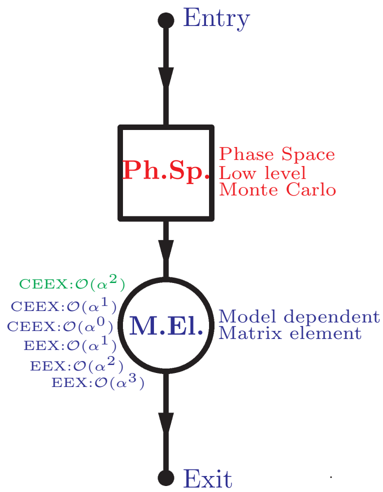
Our aims in this discussion are to summarize briefly on the main features of YFS/CEEX exponentiation [1] in QED and to present examples of recent theoretical results [2,3] relevant for the LEP/LC physics programs.

In the next section, we review the older EEX exclusive realization and summarize the new CEEX exclusive realization of the YFS [4] theory in QED. In this way we illustrate the latter's advantages over the former, which is also very successful. We also stress the key common aspects of our MC implementations of the two approaches to exponentiation, such as the exact treatment of phase space in both cases, the strict realization of the factorization theorem, etc. In Sect. 3, we illustrate recent improvements in the $\mathcal{KK}$ MC realization of CEEX for the $\nu\bar{\nu}$ channel. In Sect. 4 we illustrate recent exact results on the single hard bremsstrahlung in 2f processes which quantify the size of the missing sub-leading $\mathcal{O}(\alpha^2)$L terms in the $\mathcal{KK}$ MC. Sect. 5 contains our summary.

*Work partly supported by EU contract HPRN-CT-2000-00149, by NATO Grant PST.CLG.977751, and by Polish Government grants 5P03B09320 and 2P03B00122 and by US Department of Energy Contract DE-FG05-91ER40627.

## 2. Standard Model calculations for LEP with YFS exponentiation

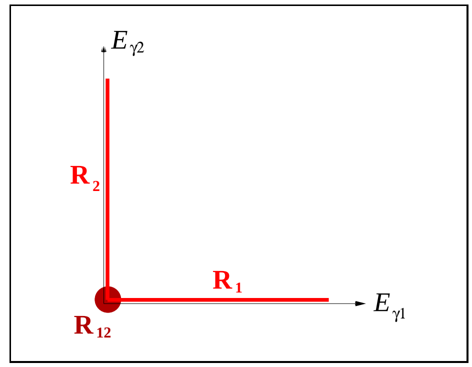
There are currently many successful applications [5] of the YFS theory of exponentiation for LEP and LC physics: (1), for $e^+e^- \to f\bar{f} + n\gamma$, $f = \tau, \mu, d, u, s, c$ there are YFS1 (1987-1989) $\mathcal{O}(\alpha^1)_{exp}$ ISR, YFS2$\in$KORALZ (1989-1990), $\mathcal{O}(\alpha^1 + h.o.LL)_{exp}$ ISR, YFS3$\in$KORALZ (1990-1998), $\mathcal{O}(\alpha^1 + h.o.LL)_{exp}$ ISR+FSR, and $\mathcal{KK}$ MC (98-02) $\mathcal{O}(\alpha^2 + h.o.LL)_{exp}$ ISR+FSR+IFI with $d\sigma/\sigma = 0.2\%$; (2), for $e^+e^- \to e^+e^- + n\gamma$ for $\theta < 6°$ there are BHLUMI 1.x, (1987-1990), $\mathcal{O}(\alpha^1)_{exp}$ and BHLUMI 2.x,4.x, (1990-1996), $\mathcal{O}(\alpha^1 + h.o.LL)_{exp}$ with $d\sigma/\sigma = 0.07\%$; (3), for $e^+e^- \to e^+e^- + n\gamma$ for $\theta > 6°$ there is BHWIDE (1994-1998), $\mathcal{O}(\alpha^1 + h.o.LL)_{exp}$ with $d\sigma/\sigma = 0.2(0.5)\%$ at the Z peak ( just off the Z peak ); (4), for $e^+e^- \to W^+W^- + n\gamma$, $W^\pm \to f\bar{f}$ there is KORALW (1994-2001); and, (5), for $e^+e^- \to W^+W^- + n\gamma$, $W^\pm \to f\bar{f}$ there is YFSWW3 (1995-2001), YFS exponentiation + Leading Pole Approximation with $d\sigma/\sigma = 0.4\%$ at LEP2 energies above the WW threshold. The typical MC realization we effect is in the form of "matrix element × exact phase space" principle, as we illustrate in the following diagram:




In practice it means:

- The universal exact Phase-space MC simulator is a separate module producing "raw events" (with importance sampling).
- The library of several types of SM/QED matrix elements which provides the "model weight" is another independent module ( the $\mathcal{KK}$MC example is shown ).
- Tau decays and hadronization come afterwards of course.

The main steps in YFS exponentiation are the reorganization of the perturbative complete $\mathcal{O}(\alpha^\infty)$ series such that IR-finite $\bar\beta$ components are isolated (factorization theorem) and the truncation of the IR-finite $\bar\beta$s to finite $\mathcal{O}(\alpha^n)$ with the attendant calculation of them from Feynman diagrams recursively. We illustrate here the respective factorization for overlapping IR divergences for the $2\gamma$ case – $R_{12}\in R_1$ and $R_{12}\in R_2$ as they are shown in the following picture:




$D_0(p_{f_1},p_{f_2},p_{f_3},p_{f_4}) = \bar\beta_0(p_{f_1},p_{f_2},p_{f_3},p_{f_4});$
$\quad p_{f_1}+p_{f_2} = p_{f_3}+p_{f_4}$
$D_1(p_f;k_1) = \bar\beta_0(p_f)\tilde S(k_1) + \bar\beta_1(p_f;k_1);$
$\quad p_{f_1}+p_{f_2} \neq p_{f_3}+p_{f_4}$
$D_2(k_1,k_2) = \bar\beta_0\tilde S(k_1)\tilde S(k_2) + \bar\beta_1(k_1)\tilde S(k_2) + \bar\beta_1(k_2)\tilde S(k_1) + \bar\beta_2(k_1,k_2).$
Note: $\bar\beta_0$ and $\bar\beta_1$ are used beyond their usual (Born and $1\gamma$) phase space. A kind of smooth "extrapolation" or "projection" is always necessary. We see that a recursive order-by-order calculation of the IR-finite $\bar\beta$s to a given fixed $\mathcal{O}(\alpha^n)$ is possible: specifically,
$\bar\beta_0(p_{f_1},p_{f_2},p_{f_3},p_{f_4}) = D_0(p_{f_1},p_{f_2},p_{f_3},p_{f_4}),$
$\bar\beta_1(p_f;k_1) = D_1(p_f;k_1) - \bar\beta_0(p_f)\tilde S(k_1),$
$\bar\beta_2(k_1,k_2) = D_2(k_1,k_2) - \bar\beta_0\tilde S(k_1)\tilde S(k_2) - \bar\beta_1(k_1)\tilde S(k_2) + \bar\beta_1(k_2)\tilde S(k_1), \ldots,$ allow such a truncation.

In the classic EEX/YFS schematically the $\beta$'s are truncated to $\mathcal{O}(\alpha^1)$, in the ISR example. For $e^-(p_1,\lambda_1)+e^+(p_2,\lambda_2)\to f(q_1,\lambda_1')+\bar f(q_2,\lambda_2')+\gamma(k_1,\sigma_1)+\ldots+\gamma(k_n,\sigma_n)$ , we have

$$\sigma = \sum_{n=0}^{\infty}\int\limits_{m_\gamma} d\Phi_{n+2}\, e^{Y(m_\gamma)} D_n(q_1,q_2,k_1,\ldots,k_n) \qquad (1)$$

with

$$\begin{aligned} &D_0 = \bar\beta_0, \qquad D_1(k_1) = \bar\beta_0\tilde S(k_1) + \bar\beta_1(k_1),\\ &D_2(k_1,k_2) = \bar\beta_0\tilde S(k_1)\tilde S(k_2) + \bar\beta_1(k_1)\tilde S(k_2)\\ &\qquad\qquad + \bar\beta_1(k_2)\tilde S(k_1) \qquad ,\\ &D_n(k_1,k_2\ldots k_n) = \bar\beta_0\tilde S(k_1)\tilde S(k_2)\ldots\tilde S(k_n)\\ &\quad +\bar\beta_1(k_1)\tilde S(k_2)\tilde S(k_3)\ldots\tilde S(k_n)\\ &\quad +\tilde S(k_1)\bar\beta_1(k_2)\tilde S(k_3)\ldots\tilde S(k_n) + \ldots\\ &\qquad +\tilde S(k_1)\tilde S(k_2)\tilde S(k_3)\ldots\bar\beta_1(k_n). \end{aligned}$$

The real soft factors are $\tilde S(k) = \sum_\sigma |\mathfrak{s}_\sigma(k)|^2 =$

$|\mathfrak{s}_+(k)|^2 + |\mathfrak{s}_-(k)|^2 = -\frac{\alpha}{4\pi^2}\left(\frac{q_1}{kq_1} - \frac{q_2}{kq_2}\right)^2$
and the IR-finite building blocks are
$\bar{\beta}_0 = (e^{-2\alpha\Re B_4}\sum_\lambda |\mathcal{M}_\lambda^{\text{Born+Virt.}}|^2)\big|_{\mathcal{O}(\alpha^1)}$, with $\lambda$ = fermion hel., $\sigma$ = photon hel., and
$\bar{\beta}_1(k) = \sum\limits_{\lambda\sigma} |\mathcal{M}_{\lambda\sigma}^{1-\text{PHOT}}|^2 - \sum\limits_{\sigma} |\mathfrak{s}_\sigma(k)|^2 \sum\limits_{\lambda} |\mathcal{M}_\lambda^{\text{Born}}|^2$.
Everything is in terms of $\sum\limits_{spin} |...|^2$ ! Distributions $< 0$ are possible for hard $2\gamma$.

The new CEEX replaces old the EEX, where both are derived from the YFS theory [4]: EEX, Exclusive EXponentiation, is very close to the original Yennie-Frautschi-Suura formulation; CEEX, Coherent EXclusive exponentiation, is an extension of the YFS theory. Its coherence makes CEEX friendly to quantum coherence among the Feynman diagrams, so that we have the complete $|\sum\limits_{diagr.}^{n} \mathcal{M}_i|^2$ rather than the often incomplete $\sum\limits_{i,j}^{n^2} \mathcal{M}_i \mathcal{M}_j{}^*$. This means be get readily the proper treatment of narrow resonances, $\gamma \oplus Z$ exchanges, $t \oplus s$ channels, ISR$\oplus$FSR, angular ordering, etc. Examples of the EEX formulation are KORALZ/YFS2, BHLUMI, BHWIDE, YFSWW, KoralW and KORALZ; the example of the CEEX formulation is $\mathcal{KK}$MC.

We illustrate CEEX schematically with the example of ISR $\mathcal{O}(\alpha^1)$ for the process $e^-(p_1,\lambda_1) + e^+(p_2,\lambda_2) \to f(q_1,\lambda_1') + \bar{f}(q_2,\lambda_2') + \gamma(k_1,\sigma_1) + ... + \gamma(k_n,\sigma_n)$. We have

$$\sigma = \sum_{n=0}^{\infty} \int\limits_{m_\gamma} d\Phi_{n+2} \sum_{\lambda,\sigma_1,\ldots,\sigma_n} |e^{\alpha B(m_\gamma)} \mathcal{M}^\lambda_{n,\sigma_1,\ldots,\sigma_n}(k_1,...,k_n)|^2,$$

$\mathcal{M}_0^\lambda = \hat{\beta}_0^\lambda$, $\lambda$=fermion helicities,
$\mathcal{M}^\lambda_{1,\sigma_1}(k_1) = \hat{\beta}_0^\lambda \mathfrak{s}_{\sigma_1}(k_1) + \hat{\beta}^\lambda_{1,\sigma_1}(k_1)$,
$\mathcal{M}^\lambda_{2,\sigma_1,\sigma_2}(k_1,k_2) = \hat{\beta}_0^\lambda \mathfrak{s}_{\sigma_1}(k_1)\mathfrak{s}_{\sigma_2}(k_2) + \hat{\beta}^\lambda_{1,\sigma_1}(k_1)\mathfrak{s}_{\sigma_2}(k_2) + \hat{\beta}^\lambda_{1,\sigma_2}(k_2)\mathfrak{s}_{\sigma_1}(k_1)$,
$\mathcal{M}^\lambda_{n,\sigma_1,\ldots\sigma_n}(k_1,...k_n) = \hat{\beta}_0^\lambda \mathfrak{s}_{\sigma_1}(k_1)\mathfrak{s}_{\sigma_2}(k_2) ...\mathfrak{s}_{\sigma_n}(k_n) + \hat{\beta}^\lambda_{1,\sigma_1}(k_1)\mathfrak{s}_{\sigma_2}(k_2)...\mathfrak{s}_{\sigma_n}(k_n) + \mathfrak{s}_{\sigma_1}(k_1)\hat{\beta}^\lambda_{1,\sigma_2}(k_2)...\mathfrak{s}_{\sigma_n}(k_n) + ... + \mathfrak{s}_{\sigma_1}(k_1)\mathfrak{s}_{\sigma_2}(k_2)...\hat{\beta}^\lambda_{1,\sigma_2}(k_2)$.

The $\mathcal{O}(\alpha^1)$ IR-finite building blocks are:
$\hat{\beta}_0^\lambda = \left(e^{-\alpha B_4}\mathcal{M}_\lambda^{\text{Born+Virt.}}\right)\big|_{\mathcal{O}(\alpha^1)}$,
$\hat{\beta}^\lambda_{1,\sigma}(k) = \mathcal{M}^\lambda_{1,\sigma}(k) - \hat{\beta}_0^\lambda \mathfrak{s}_\sigma(k)$

Everything is done in terms of $\mathcal{M}$-amplitudes! Distributions are $\geq 0$ by construction!
In $\mathcal{KK}$MC the above is done up to $\mathcal{O}(\alpha^2)$ for ISR and FSR.

The full scale CEEX $\mathcal{O}(\alpha^r)$, r=1,2, master formula for the polarized total cross section reads as follows:

$$\sigma^{(r)} = \sum_{n=0}^{\infty} \frac{1}{n!} \int d\tau_n(p_a+p_b; p_c, p_d, k_1, \ldots, k_n) \; e^{2\alpha\Re B_4} \sum_{\sigma_i,\lambda,\bar{\lambda}} \; \sum_{i,j,l,m=0}^{3} \hat{\varepsilon}_a^i \hat{\varepsilon}_b^j \sigma^i_{\lambda_a\bar{\lambda}_a} \sigma^j_{\lambda_b\bar{\lambda}_b} \; \mathfrak{M}_{\mathfrak{n}}^{(\mathfrak{r})}\begin{pmatrix} \mathfrak{p} & \mathfrak{k}_1 & \mathfrak{k}_2 & \cdots & \mathfrak{k}_{\mathfrak{n}} \\ \lambda & \sigma_1 & \sigma_2 & \cdots & \sigma_{\mathfrak{n}} \end{pmatrix} \left[ \mathfrak{M}_{\mathfrak{n}}^{(\mathfrak{r})}\begin{pmatrix} \mathfrak{p} & \mathfrak{k}_1 & \mathfrak{k}_2 & \cdots & \mathfrak{k}_{\mathfrak{n}} \\ \lambda & \sigma_1 & \sigma_2 & \cdots & \sigma_{\mathfrak{n}} \end{pmatrix} \right]^\star \sigma^l_{\bar{\lambda}_c\lambda_c} \sigma^m_{\bar{\lambda}_d\lambda_d} \hat{h}_c^l \hat{h}_c^m. \tag{2}$$

The respective CEEX amplitudes are

$$\mathfrak{M}_{\mathfrak{n}}^{(1)}\begin{pmatrix} \mathfrak{p} & \mathfrak{k}_1 & \cdots & \mathfrak{k}_{\mathfrak{n}} \\ \lambda & \sigma_1 & \cdots & \sigma_{\mathfrak{n}} \end{pmatrix} = \sum_{\wp\in\mathcal{P}} \prod_{\mathfrak{i}=1}^{\mathfrak{n}} \mathfrak{s}^{\{\wp_\mathfrak{i}\}}_{[\mathfrak{i}]} \left\{ \beta_0^{(1)}\left({}^{\mathfrak{p}}_{\lambda}; \mathfrak{X}_\wp\right) + \sum_{j=1}^{n} \frac{\beta^{(1)}_{1\{\wp_j\}}\left({}^{p\,k_j}_{\lambda\,\sigma_j}; X_\wp\right)}{\mathfrak{s}^{\{\wp_j\}}_{[j]}} \right\} \tag{3}$$

$$\mathfrak{M}_{\mathfrak{n}}^{(2)}\begin{pmatrix} \mathfrak{p} & \mathfrak{k}_1 & \cdots & \mathfrak{k}_{\mathfrak{n}} \\ \lambda & \sigma_1 & \cdots & \sigma_{\mathfrak{n}} \end{pmatrix} = \sum_{\wp\in\mathcal{P}} \prod_{\mathfrak{i}=1}^{\mathfrak{n}} \mathfrak{s}^{\{\wp_\mathfrak{i}\}}_{[\mathfrak{i}]} \times \left\{ \beta_0^{(2)}\left({}^{p}_{\lambda}; X_\wp\right) + \sum_{j=1}^{n} \frac{\beta^{(1)}_{2\{\wp_j\}}\left({}^{p\,k_j}_{\lambda\,\sigma_j}; X_\wp\right)}{\mathfrak{s}^{\{\wp_j\}}_{[j]}} + \sum_{1\leq j<l\leq n} \frac{\beta^{(2)}_{2\{\wp_j,\wp_l\}}\left({}^{p\,k_j\,k_l}_{\lambda\,\sigma_j\,\sigma_l}; X_\wp\right)}{\mathfrak{s}^{\{\wp_j\}}_{[j]}\,\mathfrak{s}^{\{\wp_l\}}_{[l]}} \right\}. \tag{4}$$

For the details see ref. [1].

The precision tags of the $\mathcal{KK}$MC are determined by comparisons with our own semi-analytical and independent MC results and by comparison with the semi-analytical results of the program ZFITTER [6]. In Fig. 1 we illustrate such comparisons, which lead to the $\mathcal{KK}$MC precision tag $d\sigma/\sigma = 0.2\%$ for example. The ISR of ZFITTER is based on the $\mathcal{O}(\alpha^2)$ result of ref. [7], while $\mathcal{KK}$MC is totally independent! See ref. [1] for a more complete discussion.

## 3. Extension of CEEX in $\mathcal{KK}$MC to the $e^+e^- \to \nu\bar{\nu}\gamma$ process

The respective tree level process is given by the Feynman diagrams in Fig. 2. As described

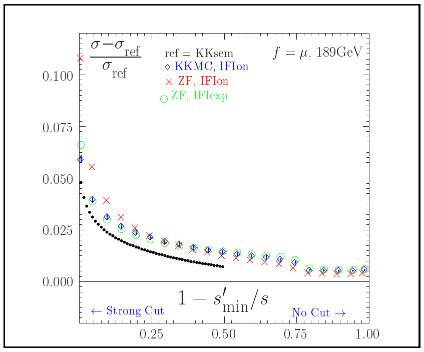



Figure 1. Cross checks of $\mathcal{KK}$MC.

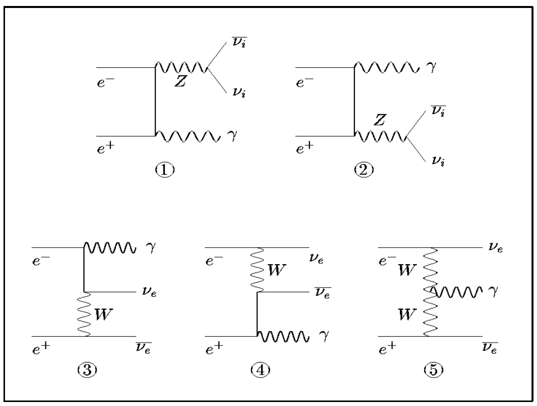



Figure 2. The process $e^+e^- \to \nu\bar{\nu}\gamma$.

in ref. [2], the $\mathcal{KK}$ MC with CEEX matrix element is now extended to the neutrino mode. It is a replacement for the older KORALZ program. This new mode of the $\mathcal{KK}$MC is useful for LEP final data analysis and for the first steps toward the LC. We note the following properties and improvements due to this new $\mathcal{KK}$MC CEEX treatment of the $e^+e^- \to \nu\bar{\nu}\gamma$ process: (1), the systematic error is now estimated to be 1.3% for $\nu_e\bar{\nu}_e\gamma$ and 0.8% for $\nu_\mu\bar{\nu}_\mu\gamma$ and $\nu_\tau\bar{\nu}_\tau\gamma$; (2), for observables with two observed photons we estimate the uncertainty to be about 5%; (3), these new improved results were obtained thanks to the inclusion of the non-photonic electroweak corrections of the `ZFITTER` package and due to newly constructed, exact, single and double photon emission amplitudes in the $\mathcal{KK}$ MC for the contribution with the $t$-channel $W$ exchange; and, (4), the virtual corrections for the $W$ exchange are at present introduced in an approximated form but the CEEX exponentiation scheme is the same as in the original $\mathcal{KK}$ MC program.

## 4. Exact Differential $\mathcal{O}(\alpha^2)$ Results for Hard Bremsstrahlung in $e^+e^- \to 2f$

The respective $\mathcal{O}(\alpha^2)$ process is illustrated in Fig. 3. In ref. [3], we have presented fully differential results for $2f+1\gamma_{virt}+1\gamma_{real}$ checked against those of Igarishi and Nakazawa in ref. [8] and partly integrated results checked against those of Berends, Burgers and Van Neerven in ref. [7]. Our results are an important component for any exact $\mathcal{O}(\alpha^2)$ exponentiated calculation for $e^+e^- \to 2f$. Similar works were also recently completed by the Karlsruhe group [9] – a comparison with their results is in progress.

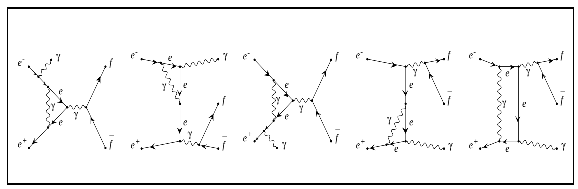


Figure 3. Representative graphs for the $1\gamma_{real} + 1\gamma_{virtual}$ correction in 2f processes.

We illustrate our results in Fig. 4. For $v < 0.9$ agreement within $0.5 \cdot 10^{-4}$ is reached.

## 5. Conclusions

YFS inspired EEX and CEEX MC schemes are successful examples of Monte Carlos based directly on the factorization theorem (albeit for the IR soft case for Abelian QED only). These

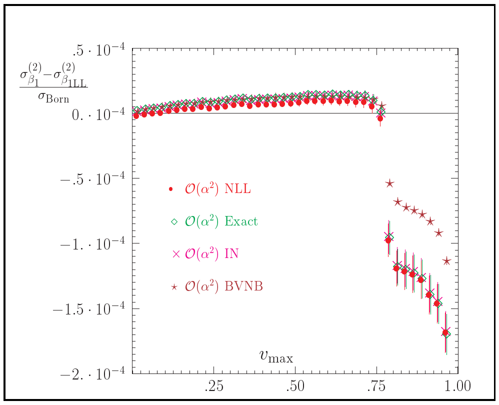



Figure 4. Exact results for the $1\gamma_{real} + 1\gamma_{virtual}$ correction in 2f processes.

schemes work well in practice: KORALZ, BHLUMI, YWSWW3, BHWIDE and $\mathcal{KK}$MC are examples. The extension of such schemes (as far as possible) to all collinear singularities would be very desirable and practically important! Work on this is in progress.

Here, we have illustrated that the $\mathcal{KK}$MC program is extended to the neutrino channel. Moreover, we have shown that the missing fully differential $2f + 1\gamma_{virt} + 1\gamma_{real}$ distributions for $\mathcal{O}(\alpha^2)$ CEEX are now available. Applications to final LEP data analysis and to LC studies are in progress.

The authors thank Profs. G. Altarelli and Wolf-Dieter Schlatter for the support and kind hospitality of the CERN Theory Division and the CERN LEP Collaborations while this work was in progress. One of the authors (B.F.L.W.) also thanks Profs. S. Bethke and L. Stodolsky for the support and kind hospitality of the Werner-Heisenberg-Institut, Max-Planck-Institut, Munich, while this work was in progress.

## REFERENCES


1. S. Jadach, B.F.L. Ward Z. Wąs, Phys. Rev. D **63** (2001) 113009; Comput. Phys. Commun. **130** (2000) 260; Eur. Phys. J. **C22** (2001) 423; Phys. Lett. **B449** (1999) 97, and references therein.
2. D. Bardin, S. Jadach, T. Riemann and Z. Was, Eur. Phys. J. **C24** (2002) 373, and references therein.
3. S. Jadach, M. Melles, B.F.L. Ward and S. A. Yost, Phys. Rev. **D65** (2002) 073030, and references therein.
4. D. R. Yennie, S. C. Frautschi, and H. Suura, Ann. Phys. **13** (1961) 379;
   see also K. T. Mahanthappa, Phys. Rev. **126** (1962) 329, for a related analysis.
5. S. Jadach and B.F.L. Ward, Phys. Rev. **D38** (1988) 2897;*ibid.* **D39** (1989) 1471; *ibid.* **D40** (1989) 3582; S.Jadach, B.F.L. Ward and Z. Was, Comput. Phys. Commun. **66** (1991) 276; S.Jadach and B.F.L. Ward, Phys. Lett. **B274** (1992) 470; S. Jadach *et al.*, Comput. Phys. Commun. **70** (1992) 305; S.Jadach, B.F.L. Ward and Z. Was, Comput. Phys. Commun. **79** (1994) 503; S. Jadach *et al.*, Phys. Lett. **B353** (1995) 362; *ibid.* **B384** (1996) 488; Comput. Phys. Commun. **102** (1997) 229; S.Jadach, W. Placzek and B.F.L. Ward, Phys. Lett. **B390** (1997) 298; Phys. Rev. **D54** (1996) 5434; Phys.Rev. D56 (1997) 6939; S.Jadach, M. Skrzypek and B.F.L. Ward, Phys. Rev. **D55** (1997) 1206; See, for example, S. Jadach *et al.*, Phys. Lett. **B417** (1998) 326; Comput. Phys. Commun. **119** (1999) 272; Phys. Rev. **D61** (2000) 113010; Phys. Rev. **D65** (2002) 093010; Comput. Phys. Commun. **140** (2001) 432, 475; S.Jadach, B.F.L. Ward and Z. Was, Comput. Phys. Commun. **124** (2000) 233; and references therein.
6. D. Bardin *et al.*, Comput. Phys. Commun. **133** (2001) 229.
7. F.A. Berends, W.L. Van Neerven and G.J.H. Burgers, Nucl. Phys. **B297** (1988) 429, and references therein.
8. M. Igarishi and N. Nakazawa, Nucl. Phys. **B288** (1987) 301.
9. H. Kuhn and G. Rodrigo, hep-ph/0204283; G. Rodrigo *et al.*, Eur. Phys. J. C**22** (2001) 81.